A Step-by-Step Example to Find Concrete Column Capacity of Arbitrary Shape, According to ACI 318-11

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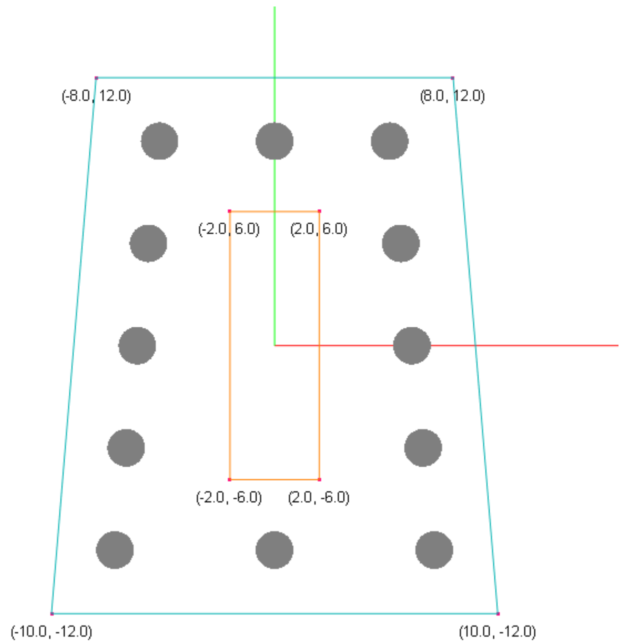
Although most of concrete columns in the world are of rectangular or round shapes, sometimes irregular-shaped concrete columns are needed for various reasons. There are a few concrete column design software tools available today that can help you do such design. However, I have not found an example that we can use to verify the accuracy of these software tools. In this article, I will present such an example using step-by-step hand calculation. It is author's hope that engineers can utilize this example to evaluate accuracy and effectiveness of any tools they might use in the design of irregular-shaped columns.

The example computes the section axial and moment capacity at a certain neutral depth. The nominal moment capacity is found by summing the moments of all the internal forces about the centroid of the section because this is the axis about which moments are computed in a conventional structural analysis. The strength reduction factor is then applied to obtain the section capacity.

Problem Statement:
Investigate the following trapezoid section with opening, according to ACI 318-2011. The section top and bottom widths are 16 and 20 inches respectively. The section height is 24 inches. The opening is 4 x 12 inches rectangle. The figure below shows the vertex coordinates of the section.
Material properties: f'_c= 6 ksi, fy = 60 ksi
Reinforcement bars: 12 #11 (diameter = 1.41 in, area = 1.56 in^2).
Concrete cover to 1.5 inches, #4 tie (diameter = 0.5 in)

Find the section capacity (φP_n and φM_n) corresponding to 50% tension at the bottom steel bars.



Solution:
Step 1: Calculate the neutral depth c at 50% tension.

The concrete cover to the center of main bars = 1.5 + 0.5 + 1.41/2 = 2.705 in.

The distance to row 5 bars from extreme compression surface d5 = 24 – 2.705 = 21.295 in.

The tensile stress in row 5 bars = 0.5 * 60 = 30 ksi.

The neutral depth $c = \frac{d}{1+f_y/(E_s*\epsilon_c)} = \frac{21.295}{1+30/(29000*0.003)} = 15.8347$ in.

Step 2: Calculate the centroid of the net section.

The whole section area $A_{whole} = \frac{16+20}{2} * 24 = 432$ in^2

Distance to the whole section centroid from base $Y_{whole} = \frac{20+16*2}{3\,(16+20)} * 24 = 11.555556$ in^2

The opening area $A_{open} = 4 * 12 = 48$ in^2

Distance to the opening centroid from base Y_{open} = 12 in.

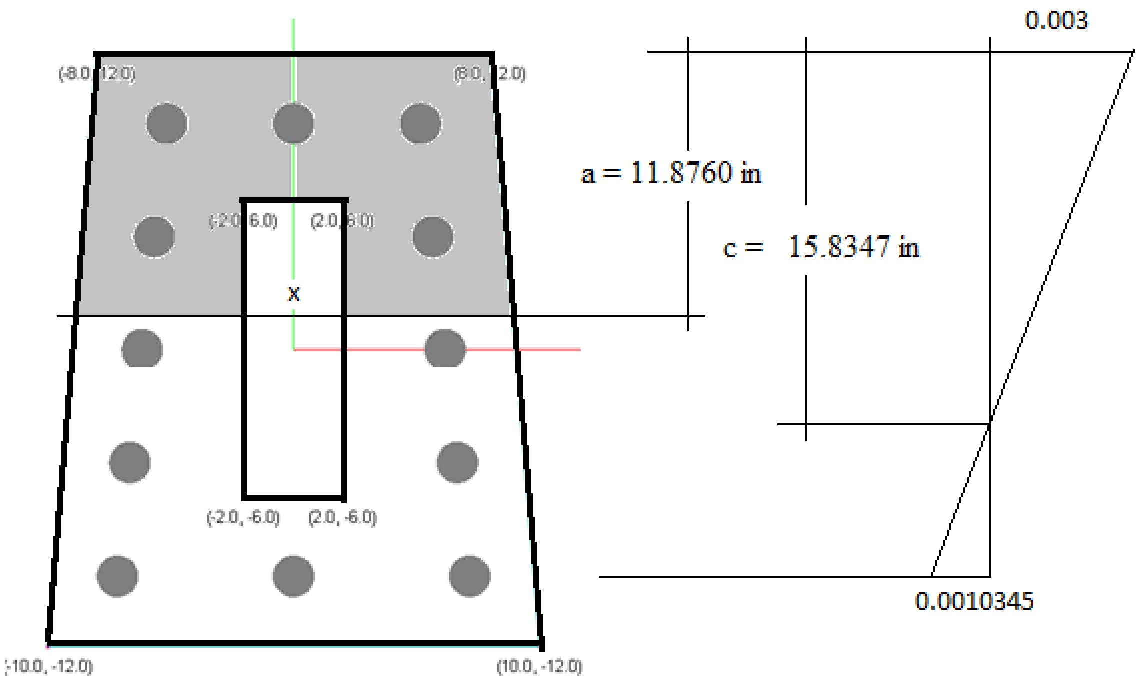
Distance to the net section centroid from the section base

$Y_{net} = \frac{-48*12+432*11.555556}{432-48} = 11.5$ in^2

Step 3: Calculate the intercepted section and opening properties at the depth a, where

$a = \beta_1 * c = 0.75 * 15.8347 = 11.8760$ in.

Intercepted trapezoid base x = $\frac{11.8760 * 2 * 2}{24} + 16 = 17.97933$ in



Intercepted trapezoid area: $A_{i\_whole} = \frac{16+17.97933}{2} * 11.8760 = 201.7693$ in^2

Distance to the centroid of the intercepted trapezoid from section top:

$$y_{i\_whole} = 11.8760 - \frac{2*16+17.97933}{3*(16+17.97933)} * 11.8760 = 6.0533 \text{ in}$$

Intercepted opening area: A_{i_open} = (11.8760 – 6) * 4 = 23.504 in^2

Distance to the centroid of the intercepted opening from section top

y_{i_open} = 6 + (11.8760 – 6) / 2 = 8.938 in

Intercepted net area $A_{i\_net} = A_{i\_whole} - A_{i\_open}$ = 201.7693 – 23.504 = 178.2653 in^2

Step 4: Calculate concrete force (C_c) and moment (M_c) about the net section centroid (distance from the section base is Y_{net} = 11.5 in).

$C_c = 0.85 * f'_c * A_{i\_net}$ = 0.85 * 6 * 178.2653 = 909.15 kips

$M_{c\_whole} = 0.85 * f'_c * A_{i\_whole}$ * (12 - 6.0533 + 0.5)

= 0.85 * 6 * 201.7693 * (12 - 6.0533 + 0.5) = 6633.8 in-kips

$M_{c\_opening} = 0.85 * f'_c * A_{i\_open}$ * (12 - 8.938 + 0.5)

= 0.85 * 6 * 23.504 * (12 – 8.938 + 0.5) = 427.0 in-kips

$M_c = M_{c\_whole} - M_{c\_opening}$ = 6633.8 – 427.0 = 6206.8 in-kips

Step 5: Calculate steel bar forces and moments about the net section centroid (distance from the section base is Y_{net} = 11.5 in). For bars located in the compression zone (d_i < a where d_i is the distance from the extreme compression fiber to bar location), the bar compressive force is reduced by compressive force of the displaced concrete (see note below).

Row	d_i (in)	Steel Strain	Steel Stress (ksi)	Steel Force F_{si} * (kips)	Distance to Net Section Centroid $\Delta y = (12 - d_i + 0.5)$ (in)	Moment $= F_{si} * \Delta y$ (in-kips)
1	2.7050	0.002487518	60.000	256.932	9.7950	2516.649
2	7.3525	0.001607015	46.603	129.491	5.1475	666.553
3	12.000	0.000726512	21.069	65.735	0.5000	32.867
4	16.6475	-0.000153991	-4.466	-13.933	-4.1475	57.788
5	21.2950	-0.001034494	-30.000	-140.402	-8.7950	1234.831
				Total **297.823**		Total **4508.689**

Note *

F_{s1} = (60 - 0.85 * 6) * 3 * 1.56 = 256.932 kips

F_{s2} = (46.603 - 0.85 * 6) * 2 * 1.56 = 129.491 kips

F_{s3} = 21.069 * 2 * 1.56 = 65.735kips

F_{s4} = -4.466* 2 * 1.56 = -13.933 kips

F_{s5} = -30 * 3 * 1.56 = -140.402 kips

Step 6: Calculate the section capacity.

P_n = 909.15 + 297.823 = 1206.97 kips; φP_n = 0.65 * 1206.97 = 784.5 kips

M_n = 6206.8 + 4508.689 = 10715.5 in-kips; φM_n = 0.65 * 10715.5 = 6965.1 in-kips

Comments: The manual computation above is verified exactly by the software tool called cColumn (see the following figure). The control point P2 in P-Mux interaction diagram corresponds to 50% tension for bottom bars.

References

1. "CRSI Handbook 2008" 10th Edition, Concrete Reinforcing Steel Institute, 2008
2. "cColumn " , Computations & Graphics, Inc., 2013

Project/Job:		cColumn V3 (C) Computations & Graphics, Inc
Engineer:	Date/Time: 05/04/13 09:49:56	

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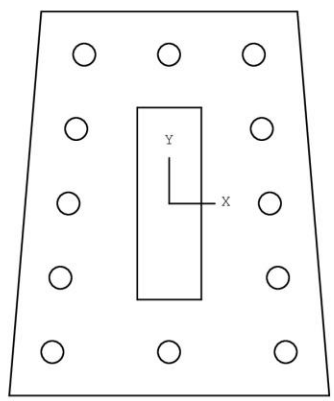

Section 1 of 1

Code: ACI 318-11
Strength reduction: Auto
Neutral axis steps: 250
Biaxial angle steps: 128
Axial capacity steps: 250
Excl. displ'd concrete: Yes

Generic
fc: 6.00 kip/in^2
fy: 60.00 kip/in^2
Cover(c.c): 2.8465 in
Bars: 12 # 11
Ag: 384.00 in^2
As: 18.72 in^2
Roh: 4.88%
Ix: 20064 in^4
Iy: 11744 in^4
Xc: 0.00 in
Yc: -0.50 in
Clear spacing: 3.26 in

Control Points (kip, kip-in)

PT	Pu	Mux(+)	max es
P0	1552.79	3542.00	-0.00070
P1	1162.72	5805.35	0.00000
P2	784.54	6965.08	0.00103
P3	479.73	7637.15	0.00207
P4	149.77	9420.48	0.00500
P5	-0.01	8685.71	0.00610

PT	Pu	Muy(-)	max es
P0	1552.78	-2589.61	-0.00066
P1	1161.24	-4488.62	0.00000
P2	815.79	-5629.83	0.00103
P3	525.68	-6286.70	0.00207
P4	98.15	-7148.53	0.00500
P5	0.01	-6773.67	0.00569

Pu-Mux and Pu-Muy Interaction Diagram